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$x_1, x_2, \dots, x_8$ , as obtained from this system of equations.

Differentiating these equations, we have respectively,

$$\left(\frac{dr_1}{d\theta_1}\right)^2, \left(\frac{dr_2}{d\theta_2}\right)^2, \dots, \left(\frac{dr_8}{d\theta_8}\right)^2 = \frac{b^2 e^4 \sin^2 \theta_1 \cos^2 \theta_1}{(1 - e^2 \cos^2 \theta_1)^3}, \frac{b^2 e^4 \sin^2 \theta_2 \cos^2 \theta_2}{(1 - e^2 \cos^2 \theta_2)^3}, \dots, \frac{b^2 e^4 \sin^2 \theta_8 \cos^2 \theta_8}{(1 - e^2 \cos^2 \theta_8)^3} \dots (2).$$

By means of the formula for the rectification of plane curves represented by polar co-ordinates, we have from (2)

$$\begin{aligned} \int_0^{\theta_1} ds_1 &= b \int_0^{\theta_1} \frac{\sqrt{[1 - e^2(2 - e^2)\cos^2 \theta_1]}}{(1 - e^2 \cos^2 \theta_1)^{\frac{3}{2}}} d\theta_1; \\ \int_0^{\theta_2} ds_2 &= b \int_0^{\theta_2} \frac{\sqrt{[1 - e^2(2 - e^2)\cos^2 \theta_2]}}{(1 - e^2 \cos^2 \theta_2)^{\frac{3}{2}}} d\theta_2; \\ \int_0^{\theta_3} ds_3 &= b \int_0^{\theta_3} \frac{\sqrt{[1 - e^2(2 - e^2)\cos^2 \theta_3]}}{(1 - e^2 \cos^2 \theta_3)^{\frac{3}{2}}} d\theta_3; \\ \int_0^{\theta_4} ds_4 &= b \int_0^{\theta_4} \frac{\sqrt{[1 - e^2(2 - e^2)\cos^2 \theta_4]}}{(1 - e^2 \cos^2 \theta_4)^{\frac{3}{2}}} d\theta_4; \\ \int_0^{\theta_5} ds_5 &= b \int_0^{\theta_5} \frac{\sqrt{[1 - e^2(2 - e^2)\cos^2 \theta_5]}}{(1 - e^2 \cos^2 \theta_5)^{\frac{3}{2}}} d\theta_5; \\ \int_0^{\theta_6} ds_6 &= b \int_0^{\theta_6} \frac{\sqrt{[1 - e^2(2 - e^2)\cos^2 \theta_6]}}{(1 - e^2 \cos^2 \theta_6)^{\frac{3}{2}}} d\theta_6; \\ \int_0^{\theta_7} ds_7 &= b \int_0^{\theta_7} \frac{\sqrt{[1 - e^2(2 - e^2)\cos^2 \theta_7]}}{(1 - e^2 \cos^2 \theta_7)^{\frac{3}{2}}} d\theta_7; \\ \int_0^{\theta_8} ds_8 &= b \int_0^{\theta_8} \frac{\sqrt{[1 - e^2(2 - e^2)\cos^2 \theta_8]}}{(1 - e^2 \cos^2 \theta_8)^{\frac{3}{2}}} d\theta_8. \end{aligned}$$

The evaluation of the thirty-two integrals indicated in (1) is a labor sufficient to discourage even a mathematical Hercules.

#### 6. Proposed by J. F. W. SCHEFFER, A. M., Hagerstown, Maryland.

Find the average length of all the diameters that can be drawn in a given ellipse.

**Solution by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in New Windsor College, New Windsor, Maryland.**

Let  $2r$  represent any diameter; then from the *central-polar* equation of

the ellipse,  $r^2 = \frac{b^2}{1 - e^2 \cos^2 \theta}$ , we have  $2r = \frac{2b}{\sqrt{1 - e^2 \cos^2 \theta}}$ ,

$$\frac{dr}{d\theta} = \frac{be^2 \sin \theta \cos \theta}{(1 - e^2 \cos^2 \theta)^{\frac{3}{2}}}; \text{ and } \frac{ds}{d\theta} = b \sqrt{\left(\frac{1 - e^2(2 - e^2)\cos^2 \theta}{(1 - e^2 \cos^2 \theta)^3}\right)}.$$

Since the number of diameters that can be drawn in an elliptic quadrant is proportional to the length of the elliptic arc bounding that quadrant, the required average length becomes

$$D=2b \int_0^{1\pi} \frac{\sqrt{[1-e^2(2-e^2)\cos^2\theta]}}{(1-e^2\cos^2\theta)^2} d\theta \div \int_0^{1\pi} \frac{\sqrt{[1-e^2(2-e^2)\cos^2\theta]}}{(1-e^2\cos^2\theta)^3} d\theta.$$

Representing  $e^2(2-e^2)$  by  $p$  and expanding,

$$\begin{aligned} D &= 2b \int_0^{1\pi} \frac{(1-\frac{1}{2}p\cos^2\theta - \frac{1}{8}p^2\cos^4\theta - \frac{1}{16}p^3\cos^6\theta - \frac{5}{128}p^4\cos^8\theta - \text{etc.})}{(1-e^2\cos^2\theta)^2} d\theta \\ &\div \int_0^{1\pi} \frac{(1-\frac{1}{2}p\cos^2\theta - \frac{1}{8}p^2\cos^4\theta - \frac{1}{16}p^3\cos^6\theta - \frac{5}{128}p^4\cos^8\theta - \text{etc.})}{(1-e^2\cos^2\theta)^3} d\theta \\ &= 2b \int_0^{1\pi} [1 + \frac{1}{2}e^2(2+e^2)\cos^2\theta + \frac{1}{8}e^4(4+12e^2-e^4)\cos^4\theta + \frac{1}{16}e^6(-8+52e^2 \\ &\quad - 10e^4+e^6)\cos^6\theta + \frac{1}{128}e^8(-272+800e^2-264e^4+56e^6-5e^8)\cos^8\theta + \text{etc.}] d\theta \\ &\div \int_0^{1\pi} [1 + \frac{1}{2}e^2(1+e^2)\cos^2\theta + \frac{1}{8}e^4(-1+10e^2-e^4)\cos^4\theta + \frac{1}{16}e^6(-15+39e^2 \\ &\quad - 9e^4+e^6)\cos^6\theta + \frac{1}{128}e^8(-261+564e^2-222e^4+28e^6-5e^8)\cos^8\theta + \text{etc.}] d\theta \\ &= 2b \left( \frac{1 + \frac{1}{2}e^2(2+e^2) + \frac{3}{4}e^4(4+12e^2-e^4) + \frac{5}{8}e^6(-8+52e^2-10e^4+e^6)}{1 + \frac{1}{2}e^2(1+e^2) + \frac{3}{4}e^4(-1+10e^2-e^4) + \frac{5}{8}e^6(-15+39e^2-9e^4+e^6)} \right), \end{aligned}$$

which is the required average length.

*Cor.*—Put  $e=\frac{1}{2}$ ; then substitute and reduce, we obtain

$$D = \frac{1218749}{1133057} \text{ of } 2b = 1.07563 \text{ times } 2b = 1.076 \times 2b = \frac{269}{250} \text{ (of the minor axis of the given ellipse).}$$

This problem was also solved by Professors *SCHIEFFER* and *ZERR*. Professor *MATZ* sent in three different solutions.

## PROBLEMS.

### 13. Proposed by I. L. BEVERAGE, Monterey, Virginia

Find the mean values of the roots of the quadratic  $x^2 - ax + b = 0$ , the roots being known to be real, but  $b$  being unknown and positive.

### 14. Proposed by CHARLES E. MYERS, Canton, Ohio.

½ of all the mellons in a patch are not ripe, and ½ of all the mellons in the same patch are rotten, the remainder being good. If a man enters the patch on a dark night and takes a melon at random, what is the probability that he will get a good one?

### 15. Proposed by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in New Windsor College, New Windsor, Maryland.

Todhunter proposes: "From a point in the circumference of a circular field a projectile is thrown at random with a given velocity, which is such that the diameter of the field is equal to the greatest range of the projectile; prove the chance of its falling within the field, is  $C=2^{-1}-2\pi^{-1}(\sqrt{2}-1), = .236+$ ." Is this result perfectly correct as to fact?